

IMPLICATIONS OF WHITE DWARF GALACTIC HALOS

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Abstract

Motivated by recent measurements which suggest that roughly half the mass of the galactic halo may be in the form of white dwarfs, we study the implications of such a halo. We first use current limits on the infrared background light and the galactic metallicity to constrain the allowed initial mass function (IMF) of the stellar population that produced the white dwarfs. The IMF must be sharply peaked about a characteristic mass scale $M_C \approx 2.3M_\odot$. Since only a fraction of the initial mass of a star is incorporated into the remnant white dwarf, we argue that the mass fraction of white dwarfs in the halo is likely to be 25% or less, and that 50% is an extreme upper limit. We use the IMF results to place corresponding constraints on the primordial initial conditions for star formation. The initial conditions must be much more homogeneous and skewed toward higher temperatures ($T_{\text{gas}} \sim 200$ K) than the conditions which lead to the present day IMF. Next we determine the luminosity function of white dwarfs. By comparing this result with the observed luminosity function, we find that the age of the halo population must be greater than ~ 16 Gyr. Finally, we calculate the radiative signature of a white dwarf halo. This infrared background is very faint, but is potentially detectable with future observations.

Subject Headings: dark matter – galaxies: structure – stars: white dwarfs – stars: evolution – stars: formation

1. INTRODUCTION

The nature of the dark matter that makes up galactic halos is an important unresolved astrophysical issue. Microlensing experiments (Alcock et al. 1993; Aubourg et al. 1993) have indicated the presence of some type of low mass stellar objects in our galactic halo. Recent measurements (MACHO collaboration 1996; see also Bennet et al. 1996) suggest that a substantial fraction (roughly half) of the halo mass is composed of white dwarfs, the remnants of an early generation of stars. In this paper, we examine the implications of a galactic halo filled with white dwarfs. We first determine necessary constraints on the distribution of masses for the stellar generation that produced these white dwarfs. We show that the resulting initial mass function (IMF) is very different from the present day IMF. In addition, its highly peaked form provides remarkable constraints on the initial conditions for star formation. We then show how a population of halo white dwarfs affects the observed luminosity function of white dwarfs. We find that in order to be consistent with the observed luminosity function of white dwarfs, the halo population is likely to have an age ~ 16 Gyr. We then use our synthesized luminosity functions to determine the radiative signature of the halo.

The idea that white dwarfs and other stellar remnants (e.g., neutron stars) are present in large quantities in galactic halos has been considered by several previous authors (e.g., Hegyi & Olive 1983, 1986, 1989; Ryu, Olive, & Silk 1990). Neutron stars are essentially ruled out because their progenitors are massive stars which leave behind too much mass in the form of heavy elements when they explode in supernovae. White dwarfs can be a viable candidate for the halo dark matter provided that the initial mass function (IMF) of the progenitor stars is confined to a narrow mass range, roughly $1 < m < 8$. Throughout this paper, we write stellar masses in solar units, i.e., we define $m \equiv M_*/(1M_\odot)$. Population synthesis models show that the bright early phases of these putative white dwarf halos can be detectable in deep galaxy counts and hence are further constrained (Charlot & Silk 1995).

The logic and organization of this paper can be summarized as follows. Using the idea that white dwarfs comprise a substantial fraction of the present day galactic halo, we find constraints on the initial mass function at the epoch of star formation in the halo (§2). This result is then used in conjunction with current theories of the IMF to constrain the physical conditions that led to star formation (§3). Next, we calculate the luminosity function of this white dwarf population (§4); we show that if sufficiently sensitive measurements of this luminosity function can be made, then the age of the halo can be cleanly determined. We determine the expected background radiation field from this galactic halo (§5). Finally, we conclude (§6) with a summary and discussion of our results.

2. THE IMPLIED IMF OF THE FIRST STELLAR GENERATION

In this section, we constrain the IMF of the stellar generation that produced the lensing white dwarfs observed in the galactic halo. For simplicity, we assume that most of the stars in the halo were produced in a single burst of star formation that occurred some time τ_H in the past. We show that the allowed IMF for this stellar distribution is highly constrained. Since current observations have placed tight limits on the mass fraction of small stars (red dwarfs) in the galactic halo, the IMF is constrained on the low mass end ($m < 1$). As we show below, the mass fraction of high mass stars ($m < 8$) is also highly constrained because these stars end their lives in supernova explosions and thereby contaminate the interstellar medium with heavy elements. The net result is that if an initial stellar population produced white dwarfs which are currently a major constituent of the galactic halo, then the IMF must be rather tightly confined to the mass range $1 < m < 8$. The remainder of this section is devoted to quantifying this assertion.

In order to proceed quantitatively, we require a description of the IMF. For the sake of definiteness, we consider the distribution $f = dN/d \ln m$ of stellar masses to be of the general log-normal form

$$\ln f(\ln m) = A - \frac{1}{2\langle\sigma\rangle^2} \left\{ \ln[m/m_C] \right\}^2, \quad (2.1)$$

where A , m_C , and $\langle\sigma\rangle$ are constants. This general form for the IMF is motivated by the current theory of star formation and by general statistical considerations (Adams & Fatuzzo 1996; see also §3; Zinnecker 1984, 1985; Larson 1973; Elmegreen & Mathieu 1983). This form for the IMF also has sufficient flexibility to assume a wide variety of behavior. The parameter A determines the overall normalization of the distribution; the parameter m_C is the mass scale (given here in solar units) which sets the center of the distribution; the parameter $\langle\sigma\rangle$ is the dimensionless width of the distribution. Notice that the shape of the distribution is completely determined by the mass scale m_C and the total width $\langle\sigma\rangle$. As a reference point, we note that if the present day IMF is fit with a log-normal form, then the shape parameters have the values $\langle\sigma\rangle \approx 1.57$ and $m_C = 0.1 - 0.2$ (see Miller & Scalo 1979; Scalo 1986; Adams & Fatuzzo 1996). As we show below, these shape parameters are highly constrained for the stellar population that filled the galactic halo with white dwarfs.

Since the age of the galaxy is $\sim 10 - 20$ Gyr, only those stars in the halo with $m > 1$ have had time to evolve into white dwarfs. Stars with smaller masses ($m < 1$) are still burning hydrogen and contributing to the infrared background light. Here we define the mass fraction \mathcal{F}_1 of the original stellar population in low mass stars to be $\mathcal{F}_1 \equiv M_{RD}/M_{TOT}$, where M_{RD} is the mass incorporated into stars in the range $m < 1$ and M_{TOT} is the total mass of the initial stellar population. Using the form (2.1) for the mass distribution, we can write this mass fraction in the form

$$\mathcal{F}_1 = \frac{M_{RD}}{M_{TOT}} = \frac{1}{2} \left\{ 1 - \text{Erf}(\xi_1) \right\}, \quad (2.2)$$

where $\text{Erf}(\xi)$ is the error function (see Abramowitz & Stegun 1970) and where the value

ξ_1 is related to the parameters in the IMF,

$$\xi_1 \equiv \frac{\sqrt{2}}{2} \left\{ \frac{\ln m_c}{\langle \sigma \rangle} + \langle \sigma \rangle \right\}. \quad (2.3)$$

For a given mass fraction \mathcal{F}_1 , we obtain a constraint on the parameters in the IMF:

$$\ln m_c + \langle \sigma \rangle^2 = \langle \sigma \rangle \sqrt{2} \operatorname{Erf}^{-1}[1 - 2\mathcal{F}_1]. \quad (2.4)$$

Similarly, the mass fraction of high mass stars is limited by metallicity considerations. We define this mass fraction to be $\mathcal{F}_2 \equiv M_{HM}/M_{TOT}$, where M_{HM} is the total mass of the initial stellar population in high mass stars with $m > 8$. The mass fraction in high mass stars can be written

$$\mathcal{F}_2 = \frac{M_{HM}}{M_{TOT}} = \frac{1}{2} \left\{ 1 - \operatorname{Erf}(\xi_2) \right\}, \quad (2.5)$$

where ξ_2 is also related to the IMF parameters and is given by

$$\xi_2 \equiv \frac{\sqrt{2}}{2} \left\{ \frac{\ln(8/m_c)}{\langle \sigma \rangle} - \langle \sigma \rangle \right\}. \quad (2.6)$$

Thus, for a given mass fraction \mathcal{F}_2 in high mass stars, we obtain a second constraint on the parameters in the IMF:

$$\ln(8/m_c) - \langle \sigma \rangle^2 = \langle \sigma \rangle \sqrt{2} \operatorname{Erf}^{-1}[1 - 2\mathcal{F}_2]. \quad (2.7)$$

For given values of the mass fractions \mathcal{F}_1 and \mathcal{F}_2 , equations (2.4) and (2.7) specify the two unknown quantities in the initial IMF. We must thus estimate the mass fractions in both low mass and high mass stars. Recent work has shown that faint red stars do not contribute significantly to the mass budget of the galactic halo (Bahcall et al. 1994; Graff & Freese 1996). This work implies that we can take $\mathcal{F}_1 \approx 0.01$ as a fairly conservative estimate. We can obtain a rough estimate of the high mass fraction \mathcal{F}_2 from a simple metallicity argument. The amount of metals (heavy elements) produced by the high mass stars in the halo is given by $\Omega_{\text{Metal}} = \Omega_{\text{Halo}} \mathcal{F}_2 f_Z$, where f_Z is the fraction of a high mass star that is ejected into the interstellar medium in the form of metals, Ω_{Halo} is the total mass density in the halo relative to the critical density of the universe, and Ω_{Metal} is the relative fraction of metals. The metallicity Z of the galactic disk is thus given by

$$Z = \frac{\Omega_{\text{Metal}}}{\Omega_{\text{Disk}}} = \frac{\Omega_{\text{Halo}}}{\Omega_{\text{Disk}}} \mathcal{F}_2 f_Z. \quad (2.8)$$

If we take the fraction of ejected metals to be $f_Z = 0.1$ and $\Omega_{\text{Halo}}/\Omega_{\text{Disk}} = 10$, then $\mathcal{F}_2 \approx Z$. As a conservative limit, we can thus take $\mathcal{F}_2 < 0.01$ (this rough argument is in good agreement with the previous results of Ryu et al. 1990; see also Hegyi & Olive

1986). Using the representative values $\mathcal{F}_1 = 0.01 = \mathcal{F}_2$, we can evaluate the constraints (2.4) and (2.7) to obtain estimates for the shape parameters in the IMF,

$$\langle \sigma \rangle = 0.44 \quad \text{and} \quad m_C = 2.3. \quad (2.9)$$

As expected, these values imply an IMF which is centered at a much higher mass scale than the present day IMF (the mass scale m_C is larger by a factor of ~ 10) and is much narrower (the width $\langle \sigma \rangle$ is smaller by a factor of ~ 3.5). The resulting IMF is shown in Figure 1 (solid curve); a fit to the present day IMF (consistent with the results of Miller & Scalo 1979) is also shown for comparison (dashed curve). Notice that the IMF at the epoch of star formation in the halo must be much more sharply peaked than that of the present day.

The derived IMF shown in Figure 1 is the mass distribution that saturates the constraints implied by equations (2.4) and (2.7). We can view this result another way by using the same constraints (this time as inequalities rather than equalities) to define an allowed region in the plane of parameters (i.e., the m_C - $\langle \sigma \rangle$ plane). The result is shown in Figure 2. In the upper left corner of the plane, we also show the point that corresponds to the parameters of the present day IMF. Figures 1 and 2 underscore the fact that the IMF of the halo population must be dramatically different from the present day IMF.

In order to determine the distribution of masses for the white dwarf population, we must specify the transformation between progenitor mass and white dwarf mass; unfortunately this relationship is somewhat uncertain (e.g., see Wood 1992 for further discussion of this issue). For this paper, we use the following transformation between progenitor mass and white dwarf mass,

$$m_{WD} = A_X \exp[B_X m], \quad (2.10)$$

with $A_X = 0.49$ and $B_X = 0.095$ (this formula is taken from the standard model of Wood 1992). For reference, we note that a progenitor star with mass $m = m_C = 2.3$ produces a white dwarf with a mass $m_{WD} \approx 0.62$. Using the IMF derived above, we can calculate the mass function of white dwarfs in the halo. The resulting mass distribution of white dwarfs in the galactic halo is shown in Figure 3. Notice that the distribution is sharply peaked about $m_{WD} \approx 0.6$. For comparison, the mass distribution of white dwarfs resulting from the present day IMF is also shown.

Since only a fraction of the progenitor mass remains in the resulting white dwarf, there is an efficiency problem associated with a galactic halo composed of white dwarfs. For the sake of definiteness, suppose that *all* of the mass in the galactic halo is efficiently processed into a stellar population with an IMF $f = dN/d \ln m$. We define the white dwarf efficiency \mathcal{E}_{WD} to be the mass fraction present in the white dwarfs resulting from the death of this initial stellar population. The value of \mathcal{E}_{WD} depends on the IMF and the relationship between progenitor mass and white dwarf mass. We can write this efficiency in the form

$$\mathcal{E}_{WD} = m_C^{-1} e^{-\langle \sigma \rangle^2/2} \frac{1}{\sqrt{2\pi}} \int_{z_1}^{z_8} dz e^{-z^2/2} m_{WD}(z), \quad (2.11)$$

where the variable $z \equiv \ln(m/m_C)/\langle\sigma\rangle$ and where the limits of integration are given by the mass range which leads to white dwarf production: $z_1 = z(m=1)$ and $z_8 = z(m=8)$. Using our derived IMF parameters (equation [2.9]) and the conversion formula (equation [2.10]), we obtain a white dwarf efficiency $\mathcal{E}_{WD} = 0.24$. Thus, even if all of the halo was incorporated into an initial stellar population, only about 1/4 of the halo mass would remain in the form of white dwarfs (for our derived IMF).

To obtain a higher white dwarf efficiency \mathcal{E}_{WD} , the IMF must be tilted toward lower masses (since the function m_{WD}/m is a monotonically decreasing function of mass). However, as the mass scale m_C in the IMF becomes smaller, the width $\langle\sigma\rangle$ must also become smaller to avoid the overproduction of red dwarfs (see Figure 2). The limiting case is thus a delta function IMF at the mass scale $m_C = 1$. In this limit, the white dwarf efficiency (eq. [2.11]) reduces to $\mathcal{E}_{WD} = m_{WD}/m = 0.54$. This value represents the maximum allowed mass fraction of white dwarfs in the halo. Notice that this efficiency constraint can be avoided if many stellar generations contribute to the halo population of white dwarfs. However, a solution involving multiple stellar generations is highly unlikely because it requires the same (very peculiar) IMF under rather different physical conditions.

Another problem associated with this low white dwarf efficiency is that a large amount of gas is left over from the process. For example, if only 1/4 of the halo mass actually becomes incorporated into white dwarfs, then the remaining 3/4 of the halo mass must reside in some other type of baryonic dark matter. Notice that only about 10% of this material can be used to make up the current disk of the galaxy. Given the difficulties associated with baryonic dark matter (Hegyi & Olive 1983, 1986), this problem is rather severe and makes the detection of large numbers of halo dwarfs (MACHO collaboration 1996) all the more startling.

3. IMPLICATIONS FOR PRIMORDIAL INITIAL CONDITIONS

In this section, we wish to examine how the current theory of star formation constrains the initial conditions during the epoch of star formation in the galactic halo. We thus require a relationship between the IMF derived in the previous section and the initial conditions. We use a recently developed theory of the IMF (Adams & Fatuzzo 1996) to obtain this relation.

Within the current paradigm of star formation (Shu, Adams, & Lizano 1987), the process which determines the IMF can be divided into two subprocesses: [1] The spectrum of initial conditions produced by the star forming environment. [2] The transformation between a particular set of initial conditions and the properties of the final (formed) star.

In the current theory, the transformation [2] is accomplished through the action of stellar winds and outflows. Stars are formed through the collapse of centrally concentrated regions in molecular clouds (cloud cores). The collapse produces a central star/disk system at the center of the flow, with material falling onto the central system at a well defined mass infall rate $\dot{M} \sim a^3/G$, where a is the effective sound speed (Shu 1977). As a nascent star gains mass, it becomes more luminous, and can produce an increasingly more powerful stellar outflow. When the strength of this outflow becomes larger than

the ram pressure of the infalling material, the star separates itself from the surrounding molecular environment and thereby determines its final mass.

The transformation between the initial conditions and the final stellar properties can be written as a “semi-empirical mass formula” (SEMF). Using the idea that the stellar mass is determined when the outflow strength exceeds the infall strength, we can write the SEMF in the form

$$L_* M_*^2 = 8m_0 \gamma^3 \delta \frac{\beta}{\alpha \epsilon} \frac{a^{11}}{G^3 \Omega^2} = \Lambda \frac{a^{11}}{G^3 \Omega^2}. \quad (3.1)$$

This formula provides us with a transformation between initial conditions (the sound speed a and the rotation rate Ω) and the final properties of the star (the luminosity L_* and the mass M_*). Furthermore, the protostellar luminosity as a function of mass is known so that equation (3.1) specifies the final stellar mass in terms of the initial conditions. In addition, the parameters α , β , γ , δ , and ϵ are efficiency factors (see Shu, Lizano, & Adams 1987; Adams & Fatuzzo 1996). In general, all of the quantities on the right hand side of equation (3.1) will have a distribution of values. These individual distributions ultimately determine the composite distribution of stellar masses M_* . However, as we argue below, to leading order the mass distribution approaches a log-normal form.

In order to evaluate the semi-empirical mass formula (3.1), we must specify the luminosity as a function of mass for young stellar objects. In general, this luminosity has many contributions (Stahler, Shu, & Taam 1980; Adams & Shu 1986; Adams 1990; Palla & Stahler 1990, 1992). In the present context, the masses of the forming stars are sharply peaked about the mass scale $m_C \approx 2.3$. For this mass range, the most important source of luminosity arises from infall, i.e., infalling material falls through the gravitational potential well of the star and converts energy into photons. The star also generates internal luminosity which becomes important at sufficiently high masses. We can parameterize these two contributions to find a luminosity versus mass relation of the form

$$\tilde{L} = L_*/(1L_\odot) = 70 \eta a_{35}^2 m + m^4, \quad (3.2)$$

where the first term arises from infall and the second term arises from internal luminosity (see Adams & Fatuzzo 1996 for further discussion). The efficiency parameter η is the fraction of the total available energy that is converted into photons. For spherical infall, all of the material reaches the stellar surface and $\eta \approx 1$. For infall which includes rotation, some of the energy is stored in the form of rotational and gravitational potential energy in the circumstellar disk; we generally expect $\eta \sim 1/2$. The sound speed a determines the mass infall rate onto the forming star/disk system according to $\dot{M} \sim a^3/G$. In equation (3.2), we have written the sound speed in dimensionless form, $a_{35} \equiv a/(0.35 \text{ km s}^{-1})$.

We want to find a relationship between the distributions of the initial variables and the resulting distribution of stellar masses (the IMF). For a given protostellar luminosity versus mass relationship, the semi-empirical mass formula can be written in the general form of a product of variables

$$M_* = \prod_{j=1}^n \alpha_j, \quad (3.3)$$

where the α_j represent the variables which determine the masses of forming stars (the sound speed a , the rotation rate Ω , etc., all taken to the appropriate powers). Each of these variables has a distribution $f_j(\alpha_j)$ with a mean value given by

$$\ln \bar{\alpha}_j = \langle \ln \alpha_j \rangle = \int_{-\infty}^{\infty} \ln \alpha_j f_j(\ln \alpha_j) d \ln \alpha_j, \quad (3.4)$$

and a corresponding variance given by

$$\sigma_j^2 = \int_{-\infty}^{\infty} \xi_j^2 f_j(\xi_j) d \xi_j. \quad (3.5)$$

In the limit of a large number n of variables, the composite distribution (i.e., the IMF) approaches a log-normal form. This behavior (see Adams & Fatuzzo 1996; Zinnecker 1984) is a direct consequence of the central limit theorem (e.g., Richtmyer 1978). As a result, as long as a large number of physical variables are involved in the star formation process, the resulting IMF must be approximately described by a log-normal form. The departure of the IMF from a purely log-normal form depends on the shapes of the individual distributions f_j . However, in the limit that the IMF can be described to leading order by a log-normal form, there are simple relationships between the distributions of the initial variables and the shape parameters m_C and $\langle \sigma \rangle$ that determine the IMF. The mass scale m_C is determined by the mean values of the logarithms of the original variables α_j , i.e.,

$$m_C \equiv \prod_{j=1}^n \exp[\langle \ln \alpha_j \rangle] \equiv \prod_{j=1}^n \bar{\alpha}_j, \quad (3.6)$$

where we have defined $\bar{\alpha}_j = \exp[\langle \ln \alpha_j \rangle]$. The dimensionless shape parameter $\langle \sigma \rangle$ of the IMF determines the width of the stellar mass distribution and is given by the sum

$$\langle \sigma \rangle^2 = \sum_{j=1}^n \sigma_j^2. \quad (3.7)$$

We can now use equations (3.6) and (3.7) to determine how the initial conditions for halo star formation must differ from star formation in present day molecular clouds. The results of the previous section show that the mass scale m_C of the halo IMF must be larger than that of the present day IMF by a factor of $\sim 10 - 20$. This difference implies that the mean values of the initial variables must be correspondingly larger so that the product (3.6) has the correct value. For example, we can consider the limit in which the effective sound speed is the most important physical variable. If we keep the mean values of all of the other variables the same as in the present day, then the mean sound speed must be in the range $\sim 0.70 - 0.90$ km/s (~ 3 times larger than present day) in order to obtain the mass scale $m_C = 2.3$. For the case in which only thermal pressure contributes to the sound speed, this range of values corresponds to gas temperatures of $T_{\text{gas}} \sim 120 - 200$ K. Temperatures in this general range can be readily obtained in a zero metallicity environment through three body cooling reactions (Palla, Stahler, & Salpeter

1983; Lepp & Shull 1983). Thus, the implied mass scale $m_C = 2.3$ is quite natural for halo star formation.

The total dimensionless width of the IMF is constrained to be quite small, with a maximum value $\langle \sigma \rangle \approx 0.44$. This result implies that the distributions of initial variables must be very narrow. In other words, the initial conditions for star formation in the halo must be *very homogeneous*. In order to quantify this statement, we again consider the sound speed to be the most important physical variable. Suppose, for example, that the sound speed varies by a factor \mathcal{F} . For the relevant mass range, the SEMF implies that $M_* \sim a^3$ (where we have combined equations [3.1] and [3.2]). Thus, the contribution of the variation in the sound speed to the total variance is given by $\sigma_a^2 = (3 \ln \mathcal{F})^2$. If the variance in the sound speed accounts for the entire variance in the mass distribution (i.e., we assume that $\sigma_a^2 = \langle \sigma \rangle^2$), then we can solve for the factor $\mathcal{F} = 1.16$. In other words, the effective sound speed can only vary by 16% in the primordial fluid that produced this generation of stars. This result, in turn, corresponds to an allowed temperature variation of only 32%. If we allow the other parameters in the SEMF to vary as well, then the sound speed and temperature are constrained to vary by even less than these amounts.

Before leaving this section, we briefly consider the idea that stellar masses are determined by the Jeans mass

$$M_J \equiv \frac{4\pi}{3} \left(\frac{\pi a^2}{G} \right)^{3/2} \rho^{-1/2}. \quad (3.8)$$

For the sound speed $a = 0.90$ km/s derived above (this value is also consistent with primordial cooling calculations), the Jeans mass is given by $M_J = 3.5 \times 10^5 M_\odot n^{-1/2}$, where n is the number density of the gas. Thus, to obtain the characteristic mass scale $m_C = 2.3$ required for the IMF, the number density must be extremely large: $n = 2 \times 10^{10}$ cm $^{-3}$. Since this value is many orders of magnitude larger than any expected density at the epoch of star formation in the galactic halo, the idea that the Jeans mass determines the mass scale of forming stars is essentially ruled out.

4. THE WHITE DWARF LUMINOSITY FUNCTION

In this section, we consider the ramifications of the posited halo white dwarfs on the observed luminosity function. In particular, we show that in order for the MACHO Collaboration's lensing result to be consistent with the present-day white dwarf luminosity function, the age of the majority of the halo dwarfs must exceed ~ 16 Gyr.

The nature of the local white dwarf luminosity function has been the focus of considerable prior effort. Schmidt (1959) was the first to point out that the star formation history of the galactic disk is written into the current observed population of cooling white dwarfs. Specifically, he noted that there should be no white dwarfs whose cooling times exceed the disk age, thus predicting a drastic falloff in number density at a specific luminosity. These ideas were quantified and extended by D'Antona and Mazzitelli (1978).

Liebert et al. (1979) demonstrated the existence of an abrupt falloff in the observed number of white dwarf stars below a luminosity $\log_{10}(L/L_\odot) \approx -4.5$ (see also Liebert

1980). In a pioneering effort, Winget et al. (1987) computed theoretical luminosity functions from the results of white dwarf cooling theory, and compared these functions with the observational data; they obtained a disk age of 9.3 ± 2.0 Gyr. More detailed studies of the white dwarf luminosity function confirmed this estimate of the disk age and showed that the result is remarkably robust when confronted with variations in the necessary input parameters (e.g., Iben & Laughlin 1989; Yuan 1989; Noh & Scalo 1990; and Wood 1992).

Several physical relations are needed to construct a model luminosity function for a population of cooling white dwarfs. The cooling curves themselves are of primary importance. Here, we ignore the complications brought on by chemical composition, and we assume that the cooling time is a function of the white dwarf mass m_{WD} and luminosity ℓ :

$$t_{cool} = t_c(\ell, m_{WD}). \quad (4.1)$$

Further, we assume a relation between the mass m_{WD} of the white dwarf and the mass m of its main-sequence progenitor:

$$m_{WD} = m_{WD}(m). \quad (4.2)$$

We also require a relation for the nuclear burning lifetime of the progenitor as a function of mass,

$$t_{MS} = t_{\text{evol}}(m), \quad (4.3)$$

as well as the IMF dN/dm . If we assume that the main-sequence precursors to the current lensing population all formed at a single time τ_H , each luminosity interval $d\ell$ centered around a particular luminosity ℓ is populated by white dwarfs of mass m_{WD} which satisfy the relation:

$$t_{\text{evol}}(m) + t_c(\ell, m_{WD}) = \tau_H. \quad (4.4)$$

Differentiating equations (4.2) and (4.4), we can write down the form of the luminosity function due to the present day population of halo white dwarfs:

$$\frac{dn}{dM_{\text{bol}}} = \frac{\ln 10}{2.5} \frac{\phi(dN/dm) t_c (\partial \log_{10} t_c / \partial \log_{10} \ell)_{m_{WD}}}{(dt_{\text{evol}}/dm) + (t_c/m_{WD})(\partial \log_{10} t_c / \partial \log_{10} m_{WD})_{\ell}(dm_{WD}/dm)}, \quad (4.5)$$

where ϕ is the normalization factor required to produce the requisite number density of white dwarfs in the solar neighborhood implied by the MACHO result. In keeping with the usual convention, we have written the luminosity function in terms of the differential magnitude dM_{bol} rather than the differential luminosity $d\ell$. Further details regarding the derivation of equation (4.5) are given in Iben & Laughlin (1989).

If we assume that the stars that gave rise to the white dwarfs formed over a period of time (rather than in a single burst), then the star formation rate $\phi(t)$ must be incorporated into a continuous formulation of the luminosity function (see Noh & Scalo (1990) for a rigorous derivation):

$$\frac{dn}{dM_{\text{bol}}} = \frac{\ln 10}{2.5} \ell \int_{M_1}^{M_2} \phi[t(m, \ell)] \frac{dN}{dm} \left(\frac{\partial t_c}{\partial \ell} \right)_{m_{WD}} dm. \quad (4.6)$$

As mentioned above, the previous efforts aimed at synthesizing white dwarf luminosity functions have all shown that the end results are relatively insensitive to most variations in the input physics. Our approach is therefore to define a standard model, explore its consequences, and then briefly discuss the secondary effects brought about by changes in the input parameters.

In light of the MACHO result, the total luminosity function of white dwarfs in the solar neighborhood should incorporate members from both the disk and the halo populations. Our synthesis program accounts for the disk population through numerical integration of equation (4.6), and for the halo population through evaluation of equation (4.5). For the disk contribution to the luminosity function, we adopt a standard model which is rather similar to the one introduced by Wood (1992). In particular, we assume the following:

- [1] The age of the disk $\tau_{\text{disk}} = 9.0$ Gyr.
- [2] A constant star formation rate $\phi(t) = \text{constant} = 5.0 \times 10^{-13} \text{ pc}^{-3} \text{ yr}^{-1}$. This value represents the star formation rate required to produce the current observed white dwarf density of $3.0 \times 10^{-3} \text{ pc}^{-3}$ for luminosities in the range $\log_{10}(L/L_{\odot}) > -4.5$.
- [3] A Salpeter IMF for the *disk* population: $dN/dm \propto m^{-2.35}$ (Salpeter 1955).
- [4] White dwarf progenitor lifetimes are given by the formula,

$$\log_{10} t_{\text{evol}} = 9.921 - 3.6648(\log_{10} m) + 1.9697(\log_{10} m)^2 - 0.9369(\log_{10} m)^3. \quad (4.7)$$

This result is taken from Iben & Laughlin (1989) who obtained the relation by extracting main sequence lifetimes from the stellar evolution calculations of a number of different authors.

- [5] The standard relationship between progenitor mass and white dwarf mass from Wood (1992). This relation is given in equation (2.10).

The most important element in a model luminosity function is the set of mass-dependent white dwarf cooling curves. Unfortunately, however, the theory of white dwarf cooling is both complicated and uncertain. In our calculations, we have used the cooling sequences of Winget et al. (1987), which span masses of $m = 0.4, 0.6, 0.8$, and 1.0 down to luminosities of $\log_{10}(L/L_{\odot}) = -5.0$. These curves are based on white dwarf models composed of pure carbon. The interiors of real white dwarfs are believed to be an admixture of carbon and oxygen, and the envelopes are either helium, or, in the case of DA dwarfs, helium and hydrogen. However, as stressed by Winget et al. (1987), the adjustments to the cooling times produced by the hydrogen-helium envelopes and the oxygen admixtures in the cores tend to cancel one another out. Hence the pure carbon models should produce a reasonable approximation to the actual cooling curves.

Because the halo population has had a long time to cool off, the cooling curves (Winget et al. 1987) must be extrapolated to luminosities below $\log_{10}(L/L_{\odot}) = -5.0$. As discussed by Wood (1992), white dwarfs dimmer than $\log_{10}(L/L_{\odot}) \approx -5.0$ are almost entirely crystallized. As a result, they are in the *Debye* regime and hence suffer from very low heat capacities, and will cool to invisibility within a finite time. Therefore, naive linear extrapolations of the cooling curves below $\log_{10}(L/L_{\odot}) = -5.0$ tend to severely

overestimate cooling times, which in turn adversely affect the number density estimates at the faint end of the luminosity function. To more accurately account for the effects of the Debye regime, we extend each cooling curve down to the value $\log_{10}(L/L_\odot) = -6.25$ using a prescription very similar to the one advocated by Wood (1992):

$$t - t_0 = A \left[1 - \left(\frac{L}{L_0} \right) \right]. \quad (4.8)$$

In equation (4.8), the reference values t_0 and L_0 are the age and luminosity of the $\log_{10}(L/L_\odot) = -5.0$ model of each mass sequence. The constant A is determined using the last two tabulated points in each of the cooling curves (from Winget et al. 1987).

Derivatives of the cooling curves at arbitrary values (m_{WD}, ℓ) with respect to both mass and luminosity are required to evaluate equations (4.5) and (4.6). Using centered differencing, we compute the derivatives at each of the tabulated points. Values for t_c , $\partial \log_{10} t_c / \partial \log_{10} \ell$, and $\partial \log_{10} t_c / \partial \log_{10} m_{WD}$ are then obtained by using bicubic splines. Interpolations are performed in the logarithm of all variables.

After being extended and interpolated, the Winget et al. (1987) cooling curves can be used to model the luminosity functions of both the halo and the disk. To build halo white dwarf luminosity functions, we again use the white dwarf progenitor lifetime given by equation (4.7) and the conversion relation of equation (2.10). The normalization factor in equation (4.5) is set at $\phi = 4.43 \times 10^{-3}$. This value corresponds to the total number of halo white dwarfs per pc³ at the solar circle, if one assumes an average white dwarf mass of $m_{WD} = 0.63$, a total halo density given by equation [5.4] below, and a total white dwarf contribution to the halo mass of 25%. Coincidentally, this value for ϕ yields a halo white dwarf number density which is slightly greater than the number density of *known* white dwarfs in the solar neighborhood (nearly all of which are certainly members of the warmer, brighter disk population). As our standard case, we adopt the log-normal IMF determined in §2, with values $m_C = 2.3$ and $\langle \sigma \rangle = 0.44$.

Using this aggregate of input physics, we compute the composite disk+halo luminosity functions. The result for our standard IMF is displayed in Figure 4a. For comparison purposes, we have plotted the observed local white dwarf luminosity function (from Liebert, Dahn, & Monet 1988). The three low luminosity points are represented by error boxes (following Wood 1992). In this representation, theoretical models which pass through all three boxes are considered to be consistent with the observed data.

The luminosity function for the disk population alone is indicated by the dotted line. The fact that this dotted line can only be distinguished from the composite (disk+halo) luminosity functions at low luminosities indicates that the disk population is entirely responsible for producing existing white dwarfs brighter than $\log_{10}(L/L_\odot) \approx -4.0$. The overall quality of our 9 Gyr disk luminosity function fit to the data is quite good. This finding is in concordance with the previous studies of the disk white dwarf population. As a side note, the marginal excess of stars observed at $\log_{10}(L/L_\odot) \approx -2.0$, has been attributed to a burst of star formation activity (Noh & Scalo 1990) which is believed to have occurred ~ 6 Gyr ago.

Our assumption of an IMF with the shape parameters $m_C = 2.3$ and $\langle \sigma \rangle = 0.44$, combined with the unchallenged existence of an observed decline in the white dwarf

luminosity function, essentially rules out halo populations which are less than ~ 16 Gyr old. Although such an age is not currently attractive from a cosmological standpoint, it stands in good agreement with the ages of the oldest globular clusters, which are thought to lie in the range 15–18 Gyr (e.g., Vandenberg 1983). Furthermore, the dating method provided by white dwarf cooling is only obliquely dependent on the stellar evolution calculations which determine the globular cluster ages. Variations in the main sequence progenitor lifetime relation have only a moderate effect on the age determination of the halo dwarf population.

Although the MACHO result suggests that roughly half of the halo mass resides in white dwarfs, we have argued that this fraction is perhaps more likely to be on the order of 25%, as required by mass limitations on a single generation of progenitor stars. Only a sustained epoch of halo star formation involving several generations can account for white dwarfs composing 50% of the halo. Nevertheless, a white dwarf halo population which has twice our assumed number density, and a time-of-formation spread of several billion years would not change the mean age estimate significantly. In order to produce the observed falloff in the luminosity function, the majority of the white dwarfs still must be older than ~ 16 Gyr.

The bounds of the lowest luminosity error box are primarily determined by two very dim white dwarfs. A measurement of the temperature T_{eff} for one of these objects (LP 701-29) indicates that this star has a radius $R \approx 0.01R_{\odot}$ and a mass $m \approx 0.6$ (Kapranidis & Liebert 1986). These values are slightly odd if the object is a member of the 9 Gyr disk population. For our choice of input physics, the *least* massive disk star whose white dwarf has had time to cool to the LP 701-29 luminosity of $\log_{10}(L/L_{\odot}) \approx -4.5$ has a mass of $m = 4.0$, and a remnant white dwarf mass of $m_{WD} = 0.72$. Winget et al. (1987) quote a dominant white dwarf mass of $m_{WD} = 0.80$ at the shortfall. On the other hand, if LP 701-29 belongs to a 16 Gyr old halo population, then its progenitor, whose dwarf now lies at the extreme bright end of the halo distribution, would have had a mass of $m = 1.04$, indicating a white dwarf mass of $m_{WD} = 0.55$.

Figures 4b and 4c chart the sequences of composite white dwarf luminosity functions which result from allowed variations in the shape parameters of the IMF. In Figure 4b, the shape parameters are $m_C = 2.5$ and $\langle \sigma \rangle = 0.30$, whereas in Figure 4c, the shape parameters are $m_C = 3.0$ and $\langle \sigma \rangle = 0.20$. Both of these alternate distributions have narrower peaks around higher masses, and hence produce considerably fewer stars of solar mass. Hence, their preponderance of high mass stars makes these IMFs less desirable in light of the already serious white dwarf efficiency problem outlined in §2. Nevertheless, even for these distributions, it is clear that the halo population must be considerably older than the disk in order to account for the observed paucity of white dwarfs at luminosities near $\log_{10}(L/L_{\odot}) \approx -4.5$. In each diagram, the heavy solid line corresponds to the luminosity function that most reasonably fits the observed data.

5. THE BACKGROUND RADIATION FIELD

In this section, we calculate the radiation signature of a galactic halo composed of white dwarfs (we follow the general formulation of Adams & Walker 1990). We first assume that at any given spatial point in the halo the mass distribution and properties of the white dwarf population are the same. The differential flux density dF_ν at frequency ν received at the earth from a particular point in the galactic halo in a particular direction is given by

$$dF_\nu = \Gamma_\nu \left(\frac{\Omega_T}{4\pi} \right) \rho ds, \quad (5.1)$$

where ρ is the mass density of the halo, Ω_T is the angular size of the observing beam, and ds is the line element along the given line of sight. The specific luminosity Γ_ν is defined such that $\Gamma_\nu/4\pi r^2$ is the flux density emitted at frequency ν per unit mass. The total observed flux density F_ν is obtained by integrating equation (5.1) along the line of sight,

$$F_\nu = \Gamma_\nu \left(\frac{\Omega_T}{4\pi} \right) \int \rho[r(s)] ds \equiv \Gamma_\nu \left(\frac{\Omega_T}{4\pi} \right) \rho_0 R \mathcal{I}(b, \ell), \quad (5.2)$$

where R is the distance from the sun to the galactic center, ρ_0 is a fiducial value of the density of the galactic halo (see eq. [5.4]), and where $\mathcal{I}(b, \ell)$ is a dimensionless integral (see eq. [5.5]) which depends on the viewing angle (given in galactic coordinates). Notice that we implicitly assume that the beam is sufficiently small to consider only a single line of sight in the integral (rather than integrating over the beam). The determination of the flux signature divides cleanly into three separate components (Adams & Walker 1990): the radiative component Γ_ν which depends only on the properties of the white dwarf population, the line of sight integral which depends on the density distribution of the galactic halo, and the solid angle Ω_T which depends on telescope properties. In the following discussion, we specify the properties of the white dwarf population and the density distribution of the galactic halo and then calculate the radiative flux.

The specific luminosity Γ_ν can be written

$$\Gamma_\nu = \frac{\int dm_{WD} (dN/dm_{WD}) 4\pi R_*^2(m_{WD}) \pi B_\nu[T_*(m_{WD})]}{\int m_{WD} dm_{WD} (dN/dm_{WD})}, \quad (5.3a)$$

where the dN/dm_{WD} is the distribution of masses of the white dwarfs which make up the halo. In the limit that the mass distribution is a delta function, the specific luminosity reduces to the simple form

$$\Gamma_\nu = 4\pi^2 R_*^2 B_\nu[T_*]/m_{WD}. \quad (5.3b)$$

The white dwarf radius $R_*(m_{WD})$ depends only on the mass m_{WD} and can be obtained from standard white dwarf models (e.g., Shapiro & Teukolsky 1983). For a given mass and age, the luminosity of a given white dwarf (and hence its stellar temperature T_*) can be determined from the considerations outlined in the previous section.

Next, we specify the mass density ρ of the galactic halo. We use a simple halo model with a spherically symmetric density distribution of the form

$$\rho(r) = \rho_0 \frac{\varpi^2}{\varpi^2 + r^2}, \quad (5.4)$$

where r is the radial distance from the galactic center, $\rho_0 \approx 1.3 \times 10^{-23} \text{ g cm}^{-3}$, and $\varpi \approx 2 \text{ kpc}$ (Binney & Tremaine 1988; Bahcall & Soneira 1980). In order to determine the halo emission, we must evaluate the non-dimensional integral

$$\mathcal{I}(b, \ell) = \frac{1}{\rho_0 R} \int_0^\infty ds \rho[r(s)] = \int_0^\infty \frac{ds}{R} \frac{\varpi^2}{\varpi^2 + r^2}, \quad (5.5)$$

where $R = 8.5 \text{ kpc}$ is the distance to the galactic center. In order to specify the direction angles, we use the galactic coordinates (b, ℓ) which correspond to a spherical coordinate system centered on the sun. The z -direction coincides with the direction of the North galactic pole, and the x -direction (the zero of the galactic longitude ℓ) coincides with the direction of the galactic center. Notice that the galactic latitude b is measured from the galactic equator (the plane of the disk) rather than from the galactic pole. If we define the quantities

$$\alpha \equiv \varpi/R \quad \text{and} \quad \mu \equiv \cos b \cos \ell,$$

then the integral \mathcal{I} can be evaluated to obtain the form

$$\mathcal{I}(b, \ell) = \frac{\alpha^2}{[1 + \alpha^2 - \mu^2]^{1/2}} \left\{ \frac{\pi}{2} + \tan^{-1} \frac{\mu}{[1 + \alpha^2 - \mu^2]^{1/2}} \right\}. \quad (5.6)$$

Notice that the radiation field emitted by galactic halos exhibits a well defined angular dependence. This modulation of the radiative signature can help distinguish observations of the halo from other possible sources of radiation (see also Gunn et al. 1978; Kephart & Weiler 1987; Adams & Walker 1990).

Using the above formulation, the IMF shape parameters given by equation (2.9), and the white dwarf mass versus progenitor mass relationship (2.10), we can determine the radiation field produced by a halo filled with white dwarfs. The result is shown in Figure 5 for assumed halo ages of $\tau_H = 10, 12, 14$, and 16 Gyr . We have normalized the curves such that *all* of the halo mass is in the form of white dwarfs. These curves should thus be scaled by the assumed mass fraction of white dwarfs in the halo. However, even for the brightest possible white dwarf halo (age of 10 Gyr and mass fraction of unity), the background radiation field (with brightness $I_\nu \sim 100 \text{ Jy/ster}$) is safely below current observational limits.

It is possible that future satellite missions (e.g., the SIRTF project currently being developed by NASA) can achieve sufficient sensitivity to either detect or rule out the brightest of these white dwarf halos. The SIRTF satellite is expected to have a sensitivity of $\sim 2000 \text{ Jy/ster}$ for each resolution element in its array camera (this value corresponds to a 3σ detection with an integration time of one hour and a wavelength in the range $2\text{--}6 \mu\text{m}$). Thus, measuring even the brightest possible white dwarf halo ($I_\nu \sim 100 \text{ Jy/ster}$) would require very long time integrations and co-adding of resolution elements.

6. SUMMARY AND DISCUSSION

In this paper, we have discussed the implications of a galactic halo which contains a substantial mass fraction of white dwarfs. Our specific results can be summarized as follows:

- [1] We have shown that the IMF of the initial stellar population is very highly constrained. As a result, the vast majority of stars from this population must lie in the mass range $1 < m < 8$ required for white dwarf production. This IMF is thus very different from the present day IMF (see Figures 1 and 2). The IMF shape parameters that saturate the constraints correspond to a mass scale $m_C \approx 2.3$ and a dimensionless width $\langle \sigma \rangle \approx 0.44$.
- [2] White dwarfs cannot make up the entire mass of the galactic halo if only a single stellar generation produced the compact objects. The white dwarf efficiency factor $\mathcal{E}_{WD} \approx 0.24$ for the IMF described in item [1]. The maximum possible white dwarf efficiency factor is $\mathcal{E}_{WD} \sim 0.5$, although it is extremely unlikely that this limit can be realized in practice. In addition, the large amount of gas left over from the process poses severe problems for a large mass fraction of white dwarfs in the galactic halo.
- [3] We have used current theoretical IMF developments in conjunction with the above limits on the IMF to constrain the initial conditions for star formation at the epoch of halo star formation. The initial conditions must be much more homogeneous than those which lead to the present day IMF. In addition, the mean values of the parameters which determine stellar masses must be skewed toward higher values. For example, the total effective sound speed (which represents the most important physical variable in the problem) must have a mean value of $a \approx 0.90$ km/s, about a factor of three larger than the value implied by the present day IMF. This sound speed corresponds to an effective temperature of $T_{\text{gas}} = 200$ K.
- [4] The white dwarf population in the galactic halo dominates the total white dwarf luminosity function for low luminosities $\log_{10}(L/L_{\odot}) < -4.5$. The shape and amplitude of the luminosity function is a rather sensitive function of the age of the halo population. As a result, existing limits on the white dwarf luminosity function imply that the age of the halo population must be larger than ~ 16 Gyr. Furthermore, if white dwarfs make up a substantial fraction of the halo mass, then the white dwarf luminosity function must turn up at luminosities just fainter than the current limits (see Figure 4).
- [5] We have determined the radiative signature of a galactic halo with a substantial mass fraction of white dwarfs (see Figure 5). The radiation field from such a halo is very faint and hence below current observational limits, but is almost detectable with future satellite missions.

The net results of microlensing experiments indicate that a substantial fraction of the halo mass resides in some type of low mass stellar objects. The only two reasonable candidates for these objects are either brown dwarfs or white dwarfs. The brown dwarf hypothesis implies a halo IMF which is sharply peaked about a very low mass scale, $m_C < 0.05$. The white dwarf hypothesis implies a halo IMF which is sharply peaked

about a larger mass scale $m_C = 2 - 3$. For comparison, the present day IMF has a mass scale $m_C = 0.1 - 0.2$ intermediate between these values and a much wider distribution (i.e., a much larger value of $\langle \sigma \rangle$). We argue that current theoretical work on star formation and the IMF strongly favors the white dwarf scenario (over the brown dwarf scenario). If stars play a role in determining their own masses through the action of stellar winds, then larger sound speeds (larger temperatures) lead to the production of higher mass stars. Since the sound speed (temperature) is expected to be higher during the epoch of halo star formation, the white dwarf scenario is more natural from the point of view of star formation theory. Furthermore, a halo with a considerable fraction of its mass in white dwarfs appears to require that the universe is older than at least 15 Gyr and provides a specific testable prediction: there should be a dramatic upturn in the white dwarf luminosity function at luminosities only slightly dimmer than those at which white dwarfs are currently being detected.

The results of this paper indicate that the most problematic issue for white dwarf halos is the large amount of gas left over from the production process. In other words, as shown in §2, the white dwarf efficiency is quite low. This leftover gas must either be locked up in other forms of baryonic dark matter or stripped out of the galaxy and into the intergalactic medium. If the observational result indicating a large mass fraction of white dwarfs is confirmed, then future theoretical work should concentrate on this important issue.

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FIGURE CAPTIONS

Figure 1. The derived IMF at the epoch of star formation in the halo (shown as the solid curve). This IMF saturates the constraints of §2 and has shape parameters $m_C = 2.3$ and $\langle \sigma \rangle = 0.44$. A fit to the present day IMF (from Miller & Scalo 1979) is also shown for comparison (dashed curve).

Figure 2. Allowed values for the shape parameters in the halo IMF. The allowed region of parameter space is shown as the hatched portion in the lower m_C - $\langle \sigma \rangle$ plane. The square symbol in the upper left part of the diagram shows the location corresponding to the present day IMF.

Figure 3. Mass distribution of white dwarfs in the halo derived from the halo star IMF (solid curve). The mass distribution of white dwarfs in the disk (derived from the present day IMF) is also shown for comparison (dashed curve).

Figure 4. Luminosity function for white dwarfs, including both the disk population (with age 9 Gyr) and the halo population (with ages varying in the range 10 – 20 Gyr). (a) White dwarf halo population arising from an IMF with our standard values of the shape parameters $m_C = 2.3$ and $\langle \sigma \rangle = 0.44$. (b) White dwarf halo population arising from an IMF with shape parameters $m_C = 2.5$ and $\langle \sigma \rangle = 0.30$. (c) White dwarf halo population arising from an IMF with shape parameters $m_C = 3.0$ and $\langle \sigma \rangle = 0.20$.

Figure 5. Radiative signature of a galactic halo composed of white dwarfs. Curves show the background infrared radiation field for a halo composed of white dwarfs with (total) ages $\tau_H = 10, 12, 14$, and 16 Gyr (from top to bottom). All curves use a log-normal IMF (with shape parameters $m_C = 2.3$ and $\langle \sigma \rangle = 0.44$) and the white dwarf/progenitor mass relationship of equation (2.10).